Math 115

Summer 2017

Lecture 14

Ch. 5 Polynomial Factoring & More

Zero-Factor Property or Zero-Product Rule

If $A \cdot B = 0$, then $A = 0$ or $B = 0$, Maybe both.

$0 \cdot (x-3) = 0$, $(x+7) \cdot 0 = 0$

Solve $(x+7)(x-3) = 0$ by Z.F.P.

$x+7=0$ or $x-3=0$

$\boxed{x=-7}$ $\boxed{x=3}$ $\Rightarrow \{-7, 3\}$

Solve $(x-5)(x+10)(2x-3)=0$ by Z.F.P.

Must be in factored form.

one side has to be zero.

$x-5=0$, $x+10=0$, $2x-3=0$

$\boxed{x=5}$ $\boxed{x=-10}$ $2x=3$, $\boxed{x=3/2}$

$\left\{-10, \frac{3}{2}, 5\right\}$

Solve $(x+7)(x-7)(4x+5)(4x-5)=0$

by Z.F.P.

↳ ① RHS = 0 ✓

② LHS must be completely factored. ✓

$x+7=0$, $x-7=0$, $4x+5=0$, $4x-5=0$

$x=-7$ $x=7$ $x=\frac{-5}{4}$ $x=\frac{5}{4}$

$\left\{\pm 7, \pm\frac{5}{4}\right\}$

How to Solve Polynomial equations:

1) Make RHS = 0

2) Factor LHS completely

3) Use Z.F.P. to find all Solutions

Solve $x^2 - 12 = x$

$x^2 - 12 - x = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x-4=0$ or $x+3=0$

$\boxed{x=4}$ $\boxed{x=-3}$

$\{-3, 4\}$

Solve $x^2 = 36 - 5x$

① Make RHS = 0 $x^2 - 36 + 5x = 0$

② Factor LHS Completely $x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

③ Use Z.F.P. to find all Solutions $x+9=0$ $x-4=0$

$\boxed{x=-9}$ $\boxed{x=4}$

$\{-9, 4\}$

Solve:

① $(3x-5)\left(\frac{2}{3}x+1\right)=0$

$3x-5=0$ $\quad$ $\frac{2}{3}x+1=0$

$x=\frac{5}{3}$ $\quad$ $x=\frac{-3}{2}$

$\left\{\frac{-3}{2}, \frac{5}{3}\right\}$

② $x^2+8x=-15$

$x^2+8x+15=0$

$(x+5)(x+3)=0$

$x+5=0$ $\quad$ $x+3=0$

$x=-5$ $\quad$ $x=-3$

$\{-5,-3\}$

③ $2x^2-5=3x$

$2x^2-5-3x=0$

$2x^2-3x-5=0$

$(2x-5)(x+1)=0$

$2x-5=0$ $\quad$ $x+1=0$

$\left\{-1, \frac{5}{2}\right\}$ $\quad$ $x=5/2$ $\quad$ $x=-1$

The Product of two consecutive integers is 20. Find all such integers.

x & $x+1$

$x(x+1)=20$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0$ $\quad$ $x-4=0$

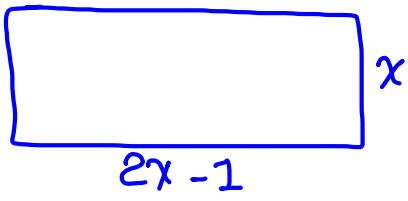
$x=-5$ $\quad$ $x=4$

x	$x+1$
4	5
-5	-4

4 & 5 or -5 & -4

The area of a rectangular pool is $45\ m^2$. Its length is 1 m shorter than twice its width.

1) Draw & label



2) Find an expression for its area

$A = LW$

$= (2x-1)x$

$= 2x^2 - x$

③ $A = 45\ m^2$

$2x^2 - x = 45$

Solve

$2x^2 - x - 45 = 0$

$P = -90$

$S = -1$

-10 & 9

-90

$2x^2 - 10x + 9x - 45 = 0$

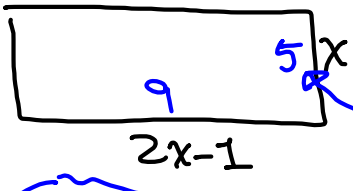
$2x(x-5) + 9(x-5) = 0$

$(x-5)(2x+9) = 0$

by Z.F.T.

$x - 5 = 0$ $\quad 2x + 9 = 0$

$x = 5$ $\quad x = -\frac{9}{2}$



Pool's dimensions

9 m by 5 m.

The sum of squares of two consecutive even integers is 100. Find all such integers.

x & $x+2$

$x \in x+2$

$x^2 \in (x+2)^2$

$x^2+(x+2)^2=100$

$x^2+(x+2)(x+2)=100$

$x^2+x^2+2x+2x+4-100=0$

$2x^2+4x-96=0$

Divide by 2

$x^2+2x-48=0$

$(x+8)(x-6)=0$

$x=-8 \quad x=6$

x	$x+2$
6	8
-8	-6

$6 \in 8$ or $-8 \in -6$

Solve:

① $(x+8)(2x-7)(2x+7)=0$

$x+8=0, \quad 2x-7=0, \quad 2x+7=0$

$x=-8 \quad x=\frac{7}{2} \quad x=\frac{-7}{2}$

$\left\{\pm\frac{7}{2}, -8\right\}$

② $4x^2-25=0$

$(2x)^2-5^2=0$

$A^2-B^2=(A+B)(A-B)$

$(2x+5)(2x-5)=0$

$x=-5/2 \quad x=5/2$

$\left\{\pm\frac{5}{2}\right\}$

③ $x(3x+1)=14$

$3x^2+x-14=0$

$P=-42$

$S=1$

-42

$7 \in -6$

$(x-2)(3x+7)=0$

$x=2 \quad x=\frac{-7}{3}$

$\left\{\frac{-7}{3}, 2\right\}$

Solve:

$49x^2(2x-5)-36(2x-5)=0$

$(7x)^2-(6)^2$

A^2-B^2

$(A+B)(A-B)$

$(2x-5)(49x^2-36)=0$

$(2x-5)(7x+6)(7x-6)=0$

$x=\frac{5}{2}$ $x=\frac{-6}{7}$ $x=\frac{6}{7}$

$\left\{\pm\frac{6}{7}, \frac{5}{2}\right\}$

Solve

$x^2+(2x-1)^2=13$

$x^2+(2x-1)(2x-1)=13$

$x^2+4x^2-2x-2x+1-13=0$

$5x^2-4x-12=0$

$P=-60$

$S=-4$

-10 & 6

-60

$5x^2-10x+6x-12=0$

$5x(x-2)+6(x-2)=0$

$(x-2)(5x+6)=0$

$x=2$ $x=\frac{-6}{5}$

$\left\{\frac{-6}{5}, 2\right\}$

Factor Completely:

1) $2x^2 - 3x - 5 = 2x^2 - 5x + 2x - 5$

$= x(2x-5) + 1(2x-5) = \boxed{(2x-5)(x+1)}$

$P = -10$

$S = -3$

$-5 \& 2$

-10

2) $64x^2 - 25$

$= (8x)^2 - 5^2$

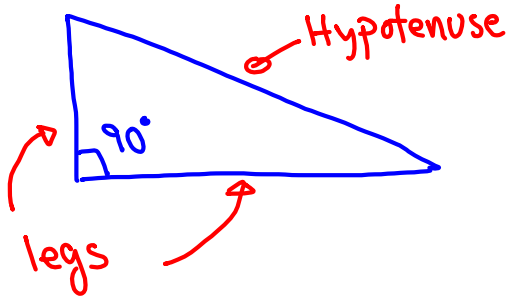
$= \boxed{(8x+5)(8x-5)}$

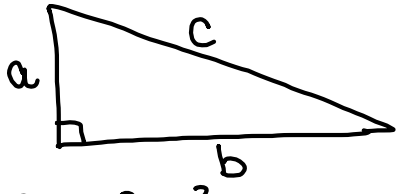
3) $8x^3 + 125$

$= (2x)^3 + 5^3$

$= \boxed{(2x+5)(4x^2 - 10x + 25)}$

Right Triangle

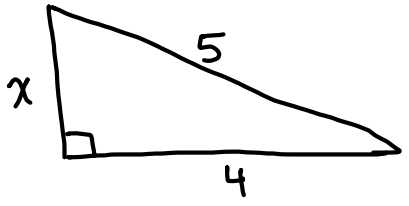




$a^2 + b^2 = c^2$

Pythagorean Thrm

Find x:



x is 3.

By Pythagorean thrm

$x^2 + 4^2 = 5^2$

$x^2 + 16 = 25$

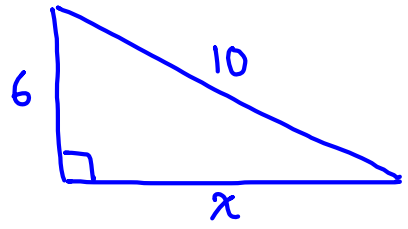
$x^2 + 16 - 25 = 0$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3$ $\quad$ $\boxed{x = 3}$

Find x:



x is 8

Right Triangle

Pythagorean thm

$6^2 + x^2 = 10^2$

$36 + x^2 = 100$

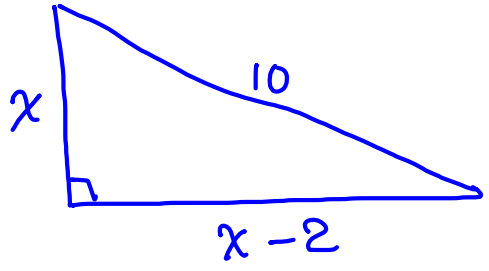
$36 + x^2 - 100 = 0$

$x^2 - 64 = 0$ $\quad x^2 - 8^2 = 0$

$(x+8)(x-8) = 0$

$x = -8$ ~~$x = -8$~~ $\quad x = 8$

Find x:



Right Triangle

Pythagorean Thm

$a^2 + b^2 = c^2$

$x^2 + (x-2)^2 = 10^2$

$x^2 + (x-2)(x-2) = 100$

$x^2 + x^2 - 2x - 2x + 4 - 100 = 0$

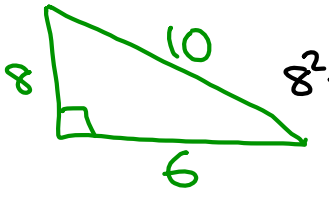
$2x^2 - 4x - 96 = 0$

Divide by 2 to reduce

$x^2 - 2x - 48 = 0$

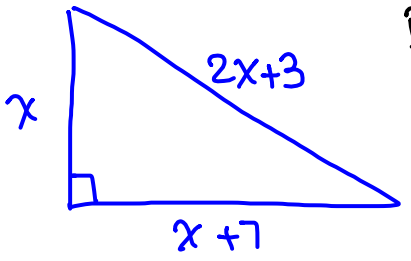
$(x-8)(x+6) = 0$

$x = 8$ ~~$x = -6$~~



$8^2 + 6^2 = 10^2$

Find x:



Right Triangle
Pythagorean Thrm

$a^2 + b^2 = c^2$

$x^2 + (x+7)^2 = (2x+3)^2$

$x^2 + (x+7)(x+7) = (2x+3)(2x+3)$

$x^2 + x^2 + 7x + 7x + 49 = 4x^2 + 6x + 6x + 9$

$2x^2 + 14x + 49 = 4x^2 + 12x + 9$

$2x^2 + 14x + 49 - 4x^2 - 12x - 9 = 0$

$-2x^2 + 2x + 40 = 0$

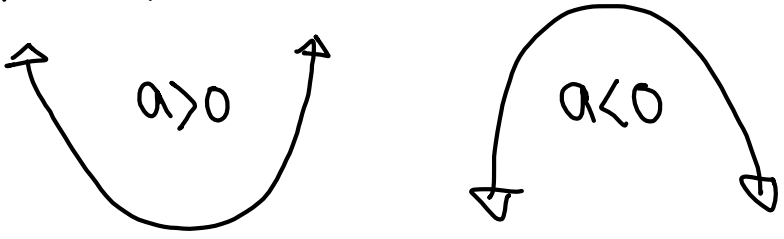
Divide by -2 to reduce

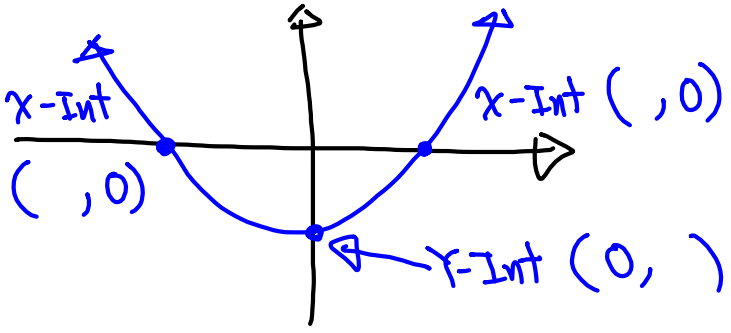
$x^2 - x - 20 = 0$

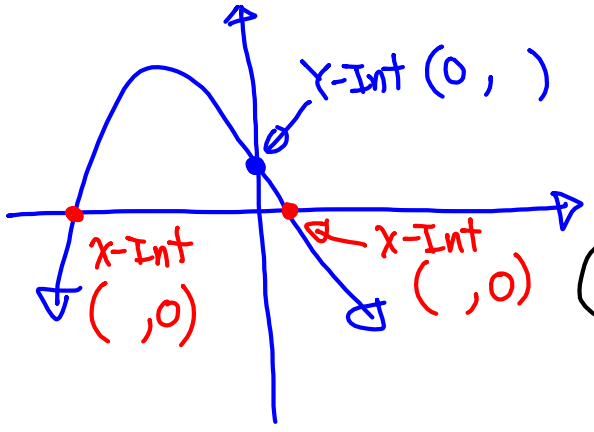
$(x-5)(x+4) = 0$

$x = 5$ ~~$x = -4$~~

Graph of $y = ax^2 + bx + c$, $a \neq 0$ is called parabola.







Find all intercepts for

$y = x^2 - x - 6$

Y-Int $(0, -6)$

x-Int $(3, 0)$ & $(-2, 0)$

$y = 0 \qquad x^2 - x - 6 = 0$

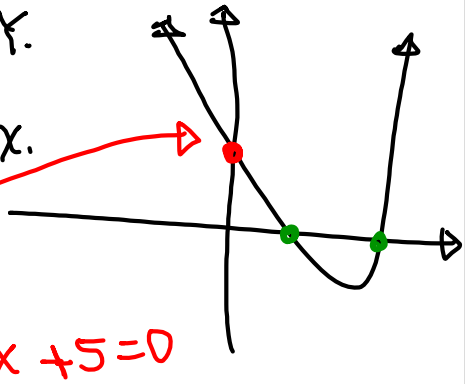
$(x-3)(x+2) = 0$

$x = 3 \qquad x = -2$

Find all intercepts for $y = 3x^2 - 8x + 5$.

Y-Int $\Rightarrow$ Let $x = 0$, find Y.

x-Int $\Rightarrow$ Let $Y = 0$, find x.



Y-Int $(0, 5)$

x-Int $\quad y = 0$

$3x^2 - 8x + 5 = 0$

$P = 15, \; S = -8$

$15 \quad -3 \; \& \; -5$

$3x^2 - 3x - 5x + 5 = 0$

$3x(x-1) - 5(x-1) = 0$

$(3x-5)(x-1) = 0$

$x = \frac{5}{3} \qquad x = 1$

$(5/3, 0), (1, 0)$

Find all intercepts for the graph of

$y = 25x^2 - 100$.

Y-Int $\rightarrow (0, -100)$

X-Ints $(2,0)$ & $(-2,0)$

X-Int

$y = 0$

$25x^2 - 100 = 0$

Divide by 25 to reduce

$x^2 - 4 = 0$

$x^2 - 2^2 = 0$

$(x-2)(x+2) = 0$

$x = 2$ $x = -2$

Square-Root method:

If $x^2 = K$, $K \geq 0$, then $x = \pm\sqrt{K}$

Solve $x^2 - 5 = 20$

$x^2 = 25$ use S.R.M.

$x = \pm\sqrt{25}$ $x = \pm 5$ $\boxed{\{\pm 5\}}$

Solve $x^2 + 4 = 44$

$x^2 = 44 - 4$

$x^2 = 40$

$x = \pm\sqrt{40}$

Can be Simplified.

$x \approx \pm 6.324$

$\{\pm\sqrt{40}\}$

Solve $(3x-5)^2=49$ by S.R.M.

$3x-5=\pm\sqrt{49}$

$3x-5=\pm 7$

$3x-5=7$ $\qquad$ $3x-5=-7$

$3x=12$ $\qquad$ $3x=-2$

$x=4$ $\qquad$ $x=\frac{-2}{3}$

$\left\{\frac{-2}{3}, 4\right\}$

Solve by S.R.M.: $(2x+7)^2=121$

$2x+7=\pm\sqrt{121}$

$2x+7=\pm 11$

$2x+7=11$ $\qquad$ $2x+7=-11$

$\boxed{x=2}$ $\qquad$ $\boxed{x=-9}$

$\{-9, 2\}$

Make a perfect-square

$x^2 + 6x + 9 = (x+3)^2$

$x^2 - 10x + 25 = (x-5)^2$

$x^2 + 15x + \frac{225}{4} = \left(x+\frac{15}{2}\right)^2$

$\frac{1}{2}\cdot 15 = \frac{15}{2}$

$x^2 - \frac{4}{5}x + \frac{4}{25} = \left(x-\frac{2}{5}\right)^2$

$\frac{1}{2}\cdot \frac{4}{5} = \frac{2}{5}$

Solve $x^2 - 10x - 24 = 0$ by Completing the square method.

$x^2 - 10x - 24 = 0$

$x^2 - 10x + 25 = 24 + 25$

$(x-5)^2 = 49$

use S.R.M. to find all Solutions

$x - 5 = \pm\sqrt{49}$

$x - 5 = \pm 7$

$x - 5 = 7$

$x = 12$

$x - 5 = -7$

$x = -2$

$\{-2, 12\}$

Solve $x^2+13x-30=0$ by Completing the sqr method.

$x^2+13x-30=0$

$x^2+13x+\frac{169}{4}=30+\frac{169}{4}$

$\frac{1}{2}\cdot 13=\frac{13}{2}$

$\left(x+\frac{13}{2}\right)^2=\frac{289}{4}$ by S.R.M.

$x+\frac{13}{2}=\pm\sqrt{\frac{289}{4}}$

$x+\frac{13}{2}=\pm\frac{17}{2}$

$x+\frac{13}{2}=\frac{17}{2}$ $\quad x=2$

$x+\frac{13}{2}=-\frac{17}{2}$ $\quad x=-15$

$\{-15,2\}$

Quadratic Eqn: $ax^2+bx+c=0;\ a\neq 0$

Quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x^2+13x-30=0$

$a=1 \quad b=13 \quad c=-30$

$b^2-4ac=(13)^2-4(1)(-30)$

$=169+120$

$=289$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\{-15,2\}$

$x=\frac{-13\pm\sqrt{289}}{2(1)}=\frac{-13\pm 17}{2}$

$x=\frac{-13+17}{2}=\frac{4}{2}=2$

$x=\frac{-13-17}{2}=\frac{-30}{2}=-15$

Solve $(2x-3)(3x+1)=7$ by Quadratic formula. Hint: FOIL & Simplify

$6x^2+2x-9x-3-7=0$

$6x^2-7x-10=0$

$a=6,\ b=-7,\ c=-10$

$b^2-4ac=(-7)^2-4(6)(-10)$

$=49+240$

$=289$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-7)\pm\sqrt{289}}{2(6)}=\frac{7\pm17}{12}$$

$x=\frac{7+17}{12}=2$ $\quad x=\frac{7-17}{12}=\frac{-5}{6}$ $\quad \left\{\frac{-5}{6},2\right\}$

The length of a rectangle with area $33\ ft^2$ is 1 ft shorter than 4 times its width. Find its dimensions.

3 ft by 11 ft

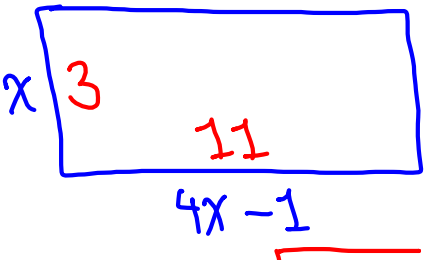
$x(4x-1)=33$

$4x^2-x-33=0$

$a=4 \quad b=-1 \quad c=-33$

$b^2-4ac=(-1)^2-4(4)(-33)$

$=529$



$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-1)\pm\sqrt{529}}{2(4)}=\frac{1\pm23}{8}$$

$x=\frac{1+23}{8}=\boxed{3}$

$x=\frac{1-23}{8}$

Agenda Monday:

1) Collect SG 16, 22, Class Project 2.

2) Lecture

3) Exam 3